THERMAL ANALYSIS OF TURBULENT POISEUILLE FLOWS

Felipe J. O. Ribeiro1, Aristeu da Silveira Neto2

1,2Federal University of Uberlandia, Av. João Naves de Ávila, 2121, Santa Mônica Campus, Uberlandia, MG

1feliperibeiro.ufu@gmail.com

2aristeus@ufu.br

***Abstract*—On the present paper the authors develop a semi-exact meta-modelled approach for thermal analysis on turbulent Poiseuille flows with a detailed study concerning the impact of different and innovative parameterizations of the turbulent Prandtl number and the Cebeci constant. The physical model consisted on a plan channel with isothermal walls that varies linearly in temperature on the stream-wise direction, resulting in an statistical permanent regime for the temperature and velocity profiles. The parametrization of the turbulent Prandtl number and the Cebeci's constant were modified aiming to achieve better accuracy when compared with the DNS solution. For the development of new models for these constants, genetic algorithms were used resulting in new functions capable of describing the turbulent Prandtl number and the Cebeci's constant for this specific problem. According to the numerical results, these new models concerning the Turbulent Prandtl number and the Cebeci's constant are capable of substantially increase the accuracy of the semi-exact method. Finally, the generally accepted classical models for these constants failed to properly represent the turbulent Poiseuille flow, what resulted in inaccuracy, this was corrected by the new models from the present research.**

***Keywords***—**Turbulent Prandtl number, Cebeci's constant, Turbulent Poiseuille flow, Genetic algorithm, DNS.**

# INTRODUCTION

It is well known that the field of turbulent fluid dynamics is capable of such high complexity that, even nowadays, it is not completely understood, as described by O. Basim and Hasan, 2007 [1]. Out of the domain of Direct Numerical Simulations (DNS), all sorts of approximations must be assumed to properly mathematically develop the Navier-Stokes equations. The nonlinear nature of these systems [2] make difficult for a continuous mathematical solution.

Despite the difficulty, a big part of the scientific community is studying turbulence on fluid and thermal dynamics. There is a great interest from the industry and academia because of opportunities for great machinery optimization and the study of chaotic dynamic behaviour.

As turbulence results in a nonlinear dynamic system, it is impossible to determine an algebraic solution for most cases. Them, the numerical approach is done, with the discretization of space and time. Solving such linear systems requires enormous computational power since the number of elements required to properly simulate the phenomenon is very big. *RANS*, *URANS* and *LES* are alternatives. They consist in not numerically resolving the Naiver-Stokes equation on all scales required, but instead substituting some tensors and other nonlinear terms by conceptual and experimental approximations.

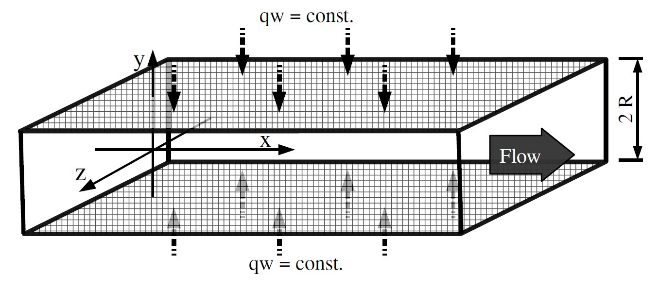
Such methods are important because they offer a much quicker solution. The DNS (Direct Numerical Solution) approach demands high computational work, as it considers all scales of complexity, not even being possible or viable in some cases [3], as explained on H. Kawamura, H. Abe and Yuichi Matsuo's work [7]. But, in the other hand, these approximated methods result in mathematical inaccuracies.

On the present paper, the authors aim to develop a semi analytical RANS (Reynold`s Averaged Navier-Stokes) method with the averaged energy equation to describe the thermal configuration on turbulent plan Poiseuille flows [8], modelled with the mixture length methodology by Ludwig Prandtl. It relies on the Boussinesq hypothesis for being a way of modelling the Reynolds shear tensor and to close the filtered Navier-Stokes and energy equations. In this context, some parameters that worth mentioning are the turbulent Prandtl number [3] and the constant , on Cebeci`s damping formulation [4]. These parameters are used for modelling important properties of the fluid`s thermal diffusion and dynamics as exposed by Silveira-Neto, 2018 [5]. Meta-models were developed for the turbulent Prandtl number and the Cebeci's constant with a genetic algorithm to enhance the results compared to DNS solutions. Such implementation resulted on accuracy and efficiency giving birth to new models for these parameters. The results of this work's formulation were compared with the DNS ([9], [10]) aiming to provide a detailed analysis on the viability of meta models on Poiseuille turbulent flows with thermal effects.

# Physical model

The problem was defined as a plan channel flow, with only one finite dimension on the axis. Two boundary walls were set at perpendicular plans to the axis, as nonslip infinite plates and in a constant thermal flux regime. The axis was proposed self-similar both on velocity and temperature, resulting on a plan domain (Fig.1). The flow was considered incompressible and the fluid was considered Newtonian. The fluid flows, in average, on the axis direction.

The Reynolds numbers ranges from to , resulting on a turbulent regime.



**Fig.1 - Geometric definition and boundary condition of the system.**

The mathematical formulation for the problem was based on the continuity and Navier-Stokes equations, as presented on Cengel's book [11], and the thermal energy transport equation, as presented on Freank's Incropera [12]. These were the assumptions made to the proposed problem, that will be considered on the differential mathematical model ahead.

# Differential Mathematical Model

The mean continuity (Eq.1), the mean Navier-Stokes (Eq.2) and the mean thermal energy (Eq.3) equations are presented as follows:

|  |  |  |
| --- | --- | --- |
| , |  | (1) |

|  |  |  |  |
| --- | --- | --- | --- |
| , |  | | (2) |
|  |  | |  |
| . | |  | (3) |

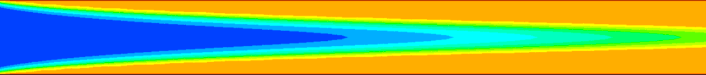
Being and the mean velocities and and the velocities´s fluctuations in and directions, the specific mass, the mean pressure, the kinematic viscosity, the mean temperature, the temperature fluctuation and the thermal diffusivity.

The time independence and mean characterization of the system define the method as an Averaged Navier Stokes (RANS) methodology.

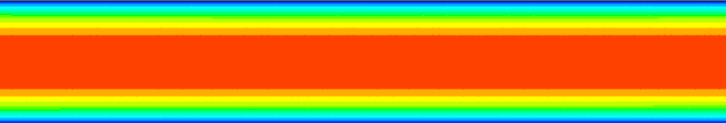
## The temperature permanent regime

The velocity field is completely developed on the channel, for mean values (Fig. 2), but this is not the case for the temperature field, as a constant thermal flux is imposed over the walls. The temperature keeps increasing in the domain, never settling down.

Even considering the mean values, the temperature field of a turbulent channel flow don't converge naturally to a unidimensional permanent state (Fig. 3).



**Fig.2 - Temperature field not in statistical permanent regime inside a channel. Profile constant in the XY plan.**



**Fig.3 - Velocity field in statistical permanent regime inside a channel. Profile constant in the XY plan.**

To simplify the solution, the thermal configuration was studied with a thermal energy balance (Eq. 4).

|  |  |  |
| --- | --- | --- |
| , |  | (4) |

where is the heat from convection, the mass flux, the specific heat capacity and is the average temperature in a cross-section.

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

where is the depth of the channel and is the heat from walls. So, substituting , where is the medium velocity in the channel's cross section and the channel half width(Fig.1), and assuming :

|  |  |  |
| --- | --- | --- |
| , |  | (6) |

|  |  |  |
| --- | --- | --- |
| , |  | (7) |

|  |  |  |
| --- | --- | --- |
| . |  | (8) |

As all terms on the right side are constants, so the mean temperature had to varies linearly on the stream-wise direction.

To better understand the wall temperature with such energy profile, a convective thermal formulation was used, which can be expressed mathematically by:

|  |  |  |
| --- | --- | --- |
| . |  | (9) |

It should be observed that the value is a constant since this is a fully dynamically developed flow. Thus, using Eq.8, it is possible to write:

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

With the temperature on the walls and the mean temperature gradient set as linear by these mathematical statements, it was possible to extend this gradient to all the domain considering the boundary conditions and the symmetry of the system. So, a constant temperature gradient was imposed on the heated walls creating a boundary condition of constant thermal flux, resulting on all the temperature field to varies linearly on the axis and with time. The temperature value was then decomposed to the form where is the temperature at the wall, resulting in a similarity effect on the streamwise direction, resuming the problem to a unidimensional representative state for the variable . Therefore, this decomposition was replaced in Eq.3:

|  |  |  |
| --- | --- | --- |
| . |  | (3) |
| . |  | (3) |

# Numerical and algebraic models proposition

The mean continuity (Eq.1), the mean Navier-Stokes (Eq.2) and the mean thermal energy (Eq.3) equations are presented as follows:

|  |  |  |
| --- | --- | --- |
| , |  | (1) |

|  |  |  |  |
| --- | --- | --- | --- |
| , |  | | (2) |
| . | |  | (3) |

Being and the mean velocities and and the velocities´s fluctuations in and directions, the specific mass, the mean pressure, the kinematic viscosity, the mean temperature, the temperature fluctuation and the thermal diffusivity.

The time independence and mean characterization of the system define the method as an Averaged Navier Stokes (RANS) methodology.

1. *Text Font of Entire Document*

The entire document should be in Times New Roman or Times font. Type 3 fonts must not be used. Other font types may be used if needed for special purposes.

Recommended font sizes are shown in Table 1.

1. Title and Author Details

Title must be in 20 pt Times New Roman font. Author name must be in 11 pt Regular font. Author affiliation must be in 10 pt Italic. Email address must be in 9 pt Courier Regular font.

TABLE I  
Font Sizes for Papers

|  |  |  |  |
| --- | --- | --- | --- |
| **Font Size** | 1. **Appearance (in Time New Roman or Times)** | | |
| **Regular** | **Bold** | **Italic** |
| 8 | table caption (in Small Caps),  figure caption,  reference item |  | reference item (partial) |
| 9 | author email address (in Courier),  cell in a table | abstract body | abstract heading (also in Bold) |
| 10 | level-1 heading (in Small Caps),  paragraph |  | level-2 heading,  level-3 heading,  author affiliation |
| 11 | author name |  |  |
| 20 | title |  |  |

All title and author details must be in single-column format and must be centered. Every word in a title must be capitalized. Email address is compulsory for the corresponding author.

1. *Section Headings*

No more than 3 levels of headings should be used. All headings must be in 10pt font. Every word in a heading must be capitalized except for short minor words as listed in Section III-B.

*Level-1 Heading*: A level-1 heading must be in Small Caps, centered and numbered using uppercase Roman numerals. For example, see heading “III. Page Style” of this document. The two level-1 headings which must not be numbered are “Acknowledgment” and “References”.

*Level-2 Heading:* A level-2 heading must be in Italic, left-justified and numbered using an uppercase alphabetic letter followed by a period. For example, see heading “C. Section Headings” above.

Level-3 Heading: A level-3 heading must be indented, in Italic and numbered with an Arabic numeral followed by a right parenthesis. The level-3 heading must end with a colon. The body of the level-3 section immediately follows the level-3 heading in the same paragraph. For example, this paragraph begins with a level-3 heading.

1. Figures and Tables

Figures and tables must be centered in the column. Large figures and tables may span across both columns. Any table or figure that takes up more than 1 column width must be positioned either at the top or at the bottom of the page.

1. Figure Captions

Figures must be numbered using Arabic numerals. Figure captions must be in 8 pt Regular font. Captions of a single line must be centered whereas multi-line captions must be justified. Captions with figure numbers must be placed after their associated figures

1. Table Captions

Tables must be numbered using uppercase Roman numerals. Table captions must be centred and in 8 pt Regular font with Small Caps. Every word in a table caption must be capitalized except for short minor words as listed in Section III-B. Captions with table numbers must be placed before their associated tables, as shown in Table 1.

1. Page Numbers, Headers and Footers

Page numbers, headers and footers must not be used.

1. Links and Bookmarks

All hypertext links and section bookmarks will be removed from papers during the processing of papers for publication. If you need to refer to an Internet email address or URL in your paper, you must type out the address or URL fully in Regular font.

References

1. Basim O. and Hasan., “*Turbulent Prandtl Number and its Use in Prediction of Heat Transfer Coefficient for liquids”*. Nahrain University, College of engineering Journal (NUCEJ) Vol.10, No.1. 2007.
2. John C. S., *Edward O. and Tamas T., “Modeling Two-Dimensional Fluid Flows with Chaos Theory”*. Johns Hopkins APL Technical Digest, volume 18, number 2, pg. 193-202*.* 1997.
3. Prandtl L. “*Uber die ausgebildete Turbulenz”*. ZAMM. 1925.
4. Cebeci T. and Bradshaw P., “*Physical and computational aspects of convective heat transfer*”. Springer, New York*.* 1984.
5. Neto A. S. “*Turbulência nos fluidos, textbook of the post graduate mechanical engineering course of federal university of Uberlandia”*. Uberlandia, Brazil.2018.
6. Kawamura H., Abe H. and Shingai, k. ,*“DNS of turbulence and heat transport in a channel flow with different Reynolds and Prandtl numbers and boundary conditions”*. Turbulence, Heat and Mass Transfer 3, (Proc. of the 3rd International Symposium on Turbulence, Heat and Mass Transfer). 2000.
7. Kawamura H., Abe H. and Yuichi M., *“DNS of turbulent heat transfer in channel flow with respect to Reynolds and Prandtl number effects”.* Elsevier, Tokio, Japan. 1999.
8. Poiseuille J. L. M., *"Recherches experimentales sur Ie mouvement des liquides dans les tubes de tres-petits diametres"*, Memoires presentes par divers savants a l'Academie Royale des Sciences de l'Institut de France, IX: 433-544. 1846.
9. Kawamura H., 2007. *“Direct Numerical Simulation Data Base for Turbulent Channel Flow with Heat Transfer”*. <*http://www.rs.tus.ac.jp/~t2lab/db/index.html*>, Laboratory of Thermo-fluid dynamics, Department of Mechanical Engineering, Faculty of Science and Technology, Tokyo University of Science, Noda-shi, Chiba-ken, Japan.
10. Kasagi H., Horiuti K., Miyake Y., Miyauchi T. and Nagano Y. 1992. *“Establishment of the Direct Numerical Simulation Data Bases of Turbulent Transport Phenomena”*. <*http://thtlab.jp/DNS/dns\\_database.html*>, Co-operative Research No. 02302043, Bunkyo-ku, Tokyo 113.
11. Cengel Y. A. and Cimbala J. M. 2006. *“Fluid mechanics Fundamentals and Applications”.* First edition, McGraw-Hill series in mechanical engineering.
12. Incropera, Freank, P., Dewitt, David, P. 2007. *“Fundamentals of Heat and Mass Transfer”.* 3rd edition, pg. 310-316, LTC, INCROPERA and DEWITT.
13. Antonialli L. A. and Silveira A. N. 2015. *“Theoretical study of fully develop turbulent flow in a flat channel, using prandtl's mixing length model”.* Journal of Applied Mathematics and Physics 06(04):677-692.
14. Price K. V., Storn M. R. and Lampinen A. J., 2005. *“Differential Evolution, a practical approach to Global Optimization”*.
15. *Section Headings*

No more than 3 levels of headings should be used. All headings must be in 10pt font. Every word in a heading must be capitalized except for short minor words as listed in Section III-B.

*Level-1 Heading*: A level-1 heading must be in Small Caps, centered and numbered using uppercase Roman numerals. For example, see heading “III. Page Style” of this document. The two level-1 headings which must not be numbered are “Acknowledgment” and “References”.

*Level-2 Heading:* A level-2 heading must be in Italic, left-justified and numbered using an uppercase alphabetic letter followed by a period. For example, see heading “C. Section Headings” above.

Level-3 Heading: A level-3 heading must be indented, in Italic and numbered with an Arabic numeral followed by a right parenthesis. The level-3 heading must end with a colon. The body of the level-3 section immediately follows the level-3 heading in the same paragraph. For example, this paragraph begins with a level-3 heading.

1. Figures and Tables

Figures and tables must be centered in the column. Large figures and tables may span across both columns. Any table or figure that takes up more than 1 column width must be positioned either at the top or at the bottom of the page.

1. Figure Captions

Figures must be numbered using Arabic numerals. Figure captions must be in 8 pt Regular font. Captions of a single line must be centered whereas multi-line captions must be justified. Captions with figure numbers must be placed after their associated figures

1. Table Captions

Tables must be numbered using uppercase Roman numerals. Table captions must be centred and in 8 pt Regular font with Small Caps. Every word in a table caption must be capitalized except for short minor words as listed in Section III-B. Captions with table numbers must be placed before their associated tables, as shown in Table 1.

1. Page Numbers, Headers and Footers

Page numbers, headers and footers must not be used.

1. Links and Bookmarks

All hypertext links and section bookmarks will be removed from papers during the processing of papers for publication. If you need to refer to an Internet email address or URL in your paper, you must type out the address or URL fully in Regular font.

References

1. Basim O. and Hasan., “*Turbulent Prandtl Number and its Use in Prediction of Heat Transfer Coefficient for liquids”*. Nahrain University, College of engineering Journal (NUCEJ) Vol.10, No.1. 2007.
2. John C. S., *Edward O. and Tamas T., “Modeling Two-Dimensional Fluid Flows with Chaos Theory”*. Johns Hopkins APL Technical Digest, volume 18, number 2, pg. 193-202*.* 1997.
3. Prandtl L. “*Uber die ausgebildete Turbulenz”*. ZAMM. 1925.
4. Cebeci T. and Bradshaw P., “*Physical and computational aspects of convective heat transfer*”. Springer, New York*.* 1984.
5. Neto A. S. “*Turbulência nos fluidos, textbook of the post graduate mechanical engineering course of federal university of Uberlandia”*. Uberlandia, Brazil.2018.
6. Kawamura H., Abe H. and Shingai, k. ,*“DNS of turbulence and heat transport in a channel flow with different Reynolds and Prandtl numbers and boundary conditions”*. Turbulence, Heat and Mass Transfer 3, (Proc. of the 3rd International Symposium on Turbulence, Heat and Mass Transfer). 2000.
7. Kawamura H., Abe H. and Yuichi M., *“DNS of turbulent heat transfer in channel flow with respect to Reynolds and Prandtl number effects”.* Elsevier, Tokio, Japan. 1999.
8. Poiseuille J. L. M., *"Recherches experimentales sur Ie mouvement des liquides dans les tubes de tres-petits diametres"*, Memoires presentes par divers savants a l'Academie Royale des Sciences de l'Institut de France, IX: 433-544. 1846.
9. Kawamura H., 2007. *“Direct Numerical Simulation Data Base for Turbulent Channel Flow with Heat Transfer”*. <*http://www.rs.tus.ac.jp/~t2lab/db/index.html*>, Laboratory of Thermo-fluid dynamics, Department of Mechanical Engineering, Faculty of Science and Technology, Tokyo University of Science, Noda-shi, Chiba-ken, Japan.
10. Kasagi H., Horiuti K., Miyake Y., Miyauchi T. and Nagano Y. 1992. *“Establishment of the Direct Numerical Simulation Data Bases of Turbulent Transport Phenomena”*. <*http://thtlab.jp/DNS/dns\\_database.html*>, Co-operative Research No. 02302043, Bunkyo-ku, Tokyo 113.
11. Cengel Y. A. and Cimbala J. M. 2006. *“Fluid mechanics Fundamentals and Applications”.* First edition, McGraw-Hill series in mechanical engineering.
12. Incropera, Freank, P., Dewitt, David, P. 2007. *“Fundamentals of Heat and Mass Transfer”.* 3rd edition, pg. 310-316, LTC, INCROPERA and DEWITT.
13. Antonialli L. A. and Silveira A. N. 2015. *“Theoretical study of fully develop turbulent flow in a flat channel, using prandtl's mixing length model”.* Journal of Applied Mathematics and Physics 06(04):677-692.
14. Price K. V., Storn M. R. and Lampinen A. J., 2005. *“Differential Evolution, a practical approach to Global Optimization”*.